Convex digital curve segmentation

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We consider the following problem. Consider the family Γ_n of all 8-connected digital curves γ_n of n pixels, where γ_n is a digitization of a closed convex curve.

Question 1 How many distinct digital curves (up to a symmetry) does Γ_n contain?

Now let κ_{γ_n} be the minimal number of linear digital segments into which a digital curve γ_n can be partitioned.

Question 2 Determine/estimate κ_{γ_n} (as a function of n and, possibly, other appropriate parameters). In particular, find/estimate κ_{γ_n} in the case when γ_n is a digitization of an ellipsis with axes a, b, $a \leq b$.

In the trivial case when γ_n is digitization of a triangle, we have $\kappa_{\gamma_n} = 3$. In another extreme case when γ_n is a digital circle, an upper bound $\kappa_{\gamma_n} = O(n^{2/3})$ follows from [1, 2]. Is that bound tight?

In the case of ellipsis with axes a and b, we conjecture that $\kappa_{\gamma_n} = \Theta(f(a, b))$, where f is some unknown function. The question is to determine/estimate the function $f(a, b)$.

References

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