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Linear-time Length Estimation in 3D Based on Minimum-length Polygonal Curves

Thomas Bülow
Computer Science Division, University of California, Berkeley
485 Soda Hall #1776, Berkeley, CA 94720-1776, USA
email: thomasbl@cs.berkeley.edu

Reinhard Klette
CITR, The University of Auckland, Tamaki campus
Morrin Road, Glen Innes, Auckland 1005, New Zealand
email: r.klette@auckland.ac.nz

The calculation of the length of a simple digital curve in three-dimensional Euclidean space was a subject in [3] by adding weights of local steps. The weights had been optimized for digital curves in 3D, but the method may produce errors for specific curves. Local methods fail to be multigrid-convergent length estimators of digitized curves.

Digital curves in 3D may be represented in different ways. One option is to consider simple closed one-dimensional grid-continua in the 3D orthogonal grid defined by sequences of grid cubes such that each grid cube in the sequence has exactly two face-adjacent grid cubes in this sequence, and the union of all cubes of the sequence is homeomorphic to a torus [5]. The length of such a digital curve may be estimated by the length of a shortest polygonal curve contained in this union of cubes and not contractible into a single point in this union of cubes. This length estimator is multigrid convergent for digitized smooth convex curves [4]. Iterative algorithms for approximating such shortest polygonal curves in 3D have been discussed in [1], with experimentally measured linear time complexity. However, the open question remains:

Is there a linear-time (on-line or off-line) algorithm for calculating such shortest polygonal curves for multigrid-convergent estimation of the length of a digitized 3D curve?

In [2] it has been shown that 3D digital straight segment approximation allows multigrid-convergent length estimation, and a linear-time algorithm is also available for this method. In case the open problem can be positively solved then a comparison with the straight segment approximation method of [2] would be the logical consequence.

References

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