Open Problems

in (Digital) Geometry
- collected during one week at Dagstuhl in March 2004 -
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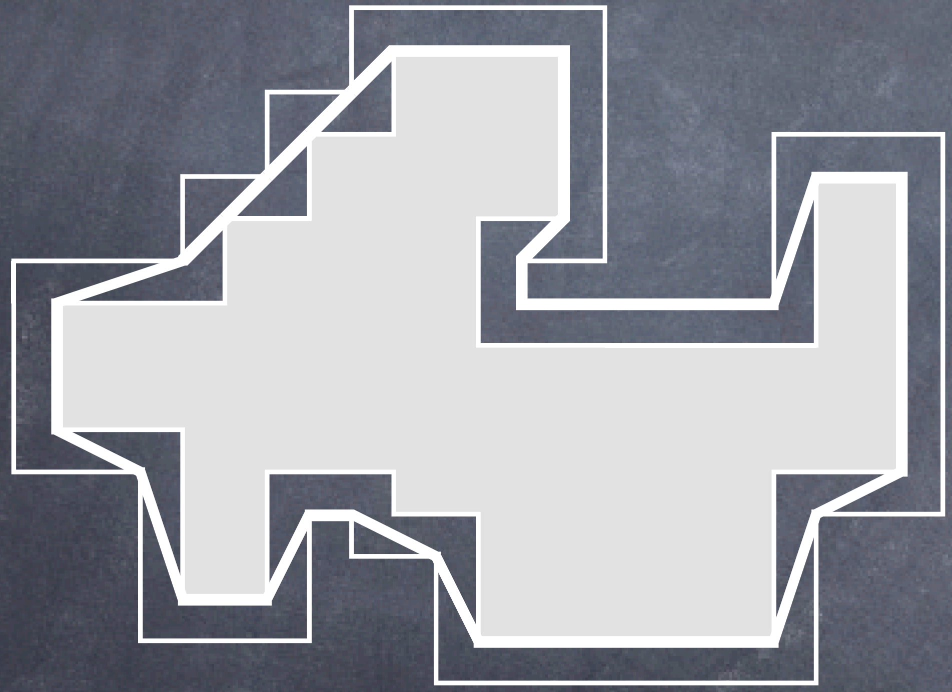
(thanks to all who contributed to the list)

Problem 1: Relative Convex Hull in 3D

Let S be a subset of B. S is called B-convex iff, for all p,q in S, if the straight line segment pq is in B, it is also in S. The B-convex hull of S is the intersection of all B-convex sets that contain S.



Calculate the relative convex hull for two simple polyhedra A and B in 3D space. Note: 2D algorithm is not hard (for two simple polygons).

Problem 2: MLP in simple 2-curves of grid cubes

The vertices of a minimum-length polygon contained (and not contractible into a single point) in a simple 2-curve of grid cubes are limited to be on critical edges (incident with three cubes in the curve) only.

Provide an (efficient) algorithm for calculating such MLPs.

Problem 3: Cycles of Median Operators

Repeatedly apply a median operator to a finite or infinite binary picture (in parallel), generating a sequence of binary pictures. Characterize median operators which may produce a cycle (of length 2 or more) on finite pictures.

x	x	x		0	0	1	1	0	0	1	1
x	x	x		1	1	0	0	1	1	0	0
x	x	x		0	0	1	1	0	0	1	1

Problem 4: Bounds for perimeters defined by moments

Grid point theory allows a (trivial) lower bound for perimeters of convex sets defined by the discrete moment $m_{0,0}$.

Specify bounds for perimeters of planar sets defined by discrete moments (of arbitrary order).

Problem 5: Number of digital topologies in Z^n

All connected sets are 0-connected, all disconnected sets are (n-1)-disconnected, and the closure of any singleton is (n-1)-connected.

in 2D: two (alternating topology, grid-cell topology)

in 3D: five

in 4D: 24

...

in nD: ???

Problem 6: axiomatic theory of good pairs

0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	1	0	0
0	1	0	0	0	1	0
0	0	1	0	1	0	0
0	0	0	1	0	0	0
0	0	0	0	0	0	0

2D: YES - (4,8), (6,6), (8,4); NO - (4,4), (8,8)

3D: YES - (6,26), (6,18), (18,6), (26,6);

NO - (6,6), (18,18), (18,26), (26,26), ...

Problem 7: periodicity of binary patterns

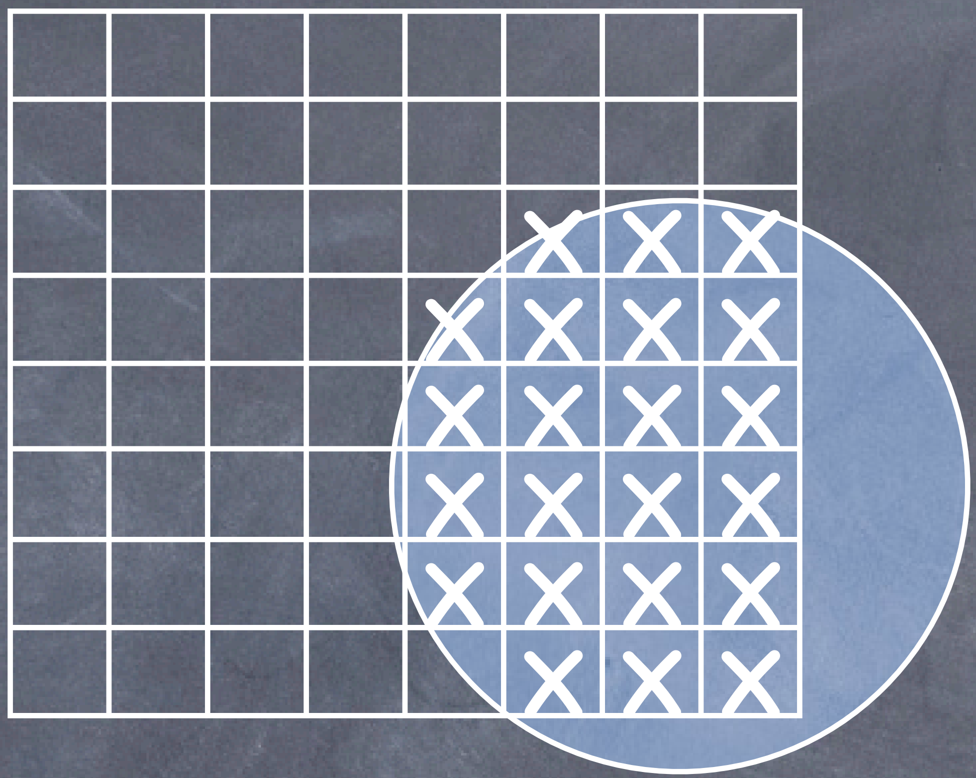
0	0		0	0	0	0	0	0	0	0	0	0	0
0	1		0	1	0	1	0	1	0	1	0	1	0
			0	0	0	0	0	0	0	0	0	0	0
1	0		1	0	1	0	1	0	1	0	1	0	1
0	0		0	0	0	0	0	0	0	0	0	0	0
			1	0	1	0	1	0	1	0	1	0	1
0	1		0	0	0	0	0	0	0	0	0	0	0
0	0		1	0	1	0	1	0	1	0	1	0	1
			0	0	0	0	0	0	0	0	0	0	0
0	0		0	1	0	1	0	1	0	1	0	1	0
1	0		0	0	0	0	0	0	0	0	0	0	0

conjecture: if the number of different configurations in hxl windows is less than or equal to hl, then the infinite binary pattern is periodic (into at least one direction)

Problem 8: number of hxl-configurations in a naive digital plane

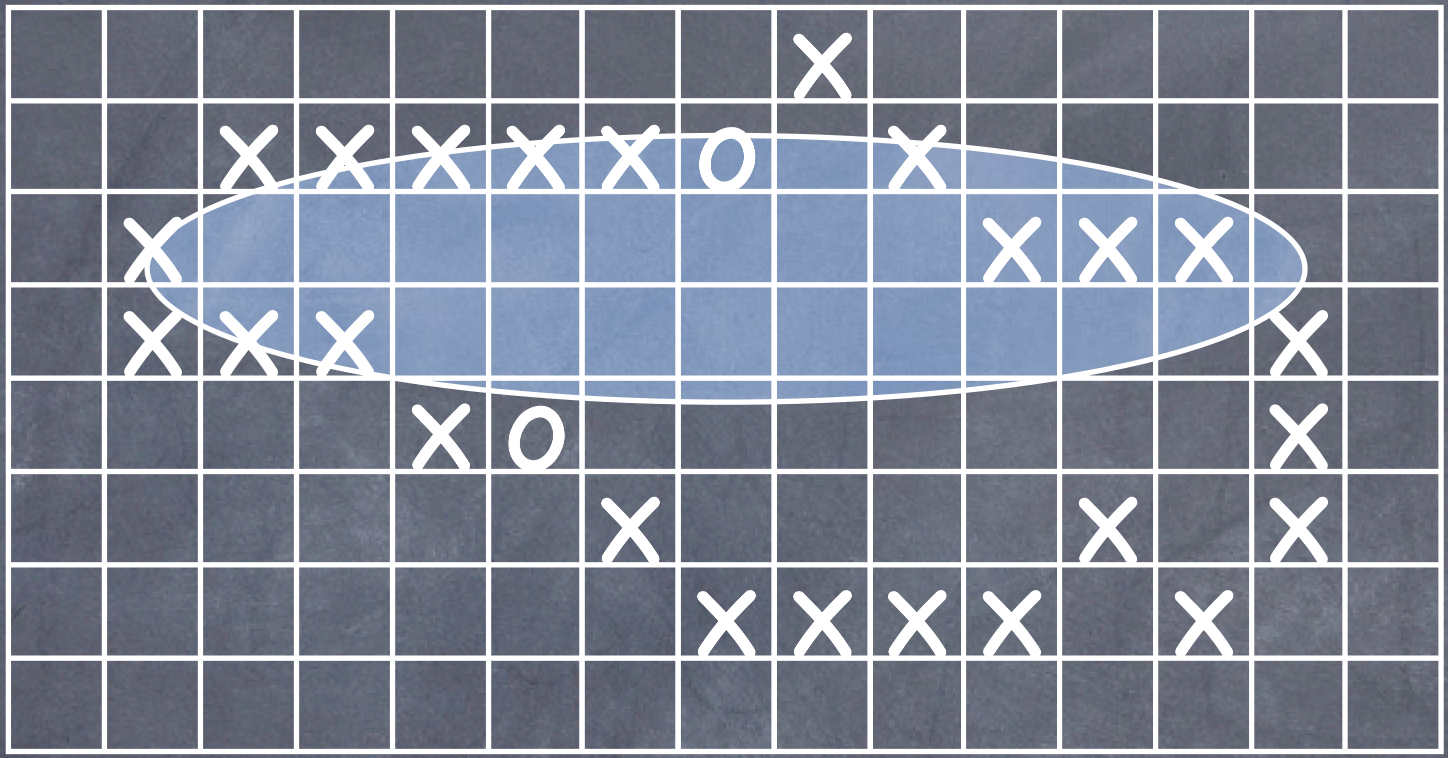
Assume a digital plane where each grid point (x,y) has exactly one grid point (x,y,z) in that plane. Consider configurations of z values in hxl windows in the xy-plane (relatively to a fixed reference point in these windows). What is the total number of different hxl configurations (for any naive digital plane)? For example, for h=l=2 we have 5 (and at most 4 in one plane).

Problem 9: Number of disk segments



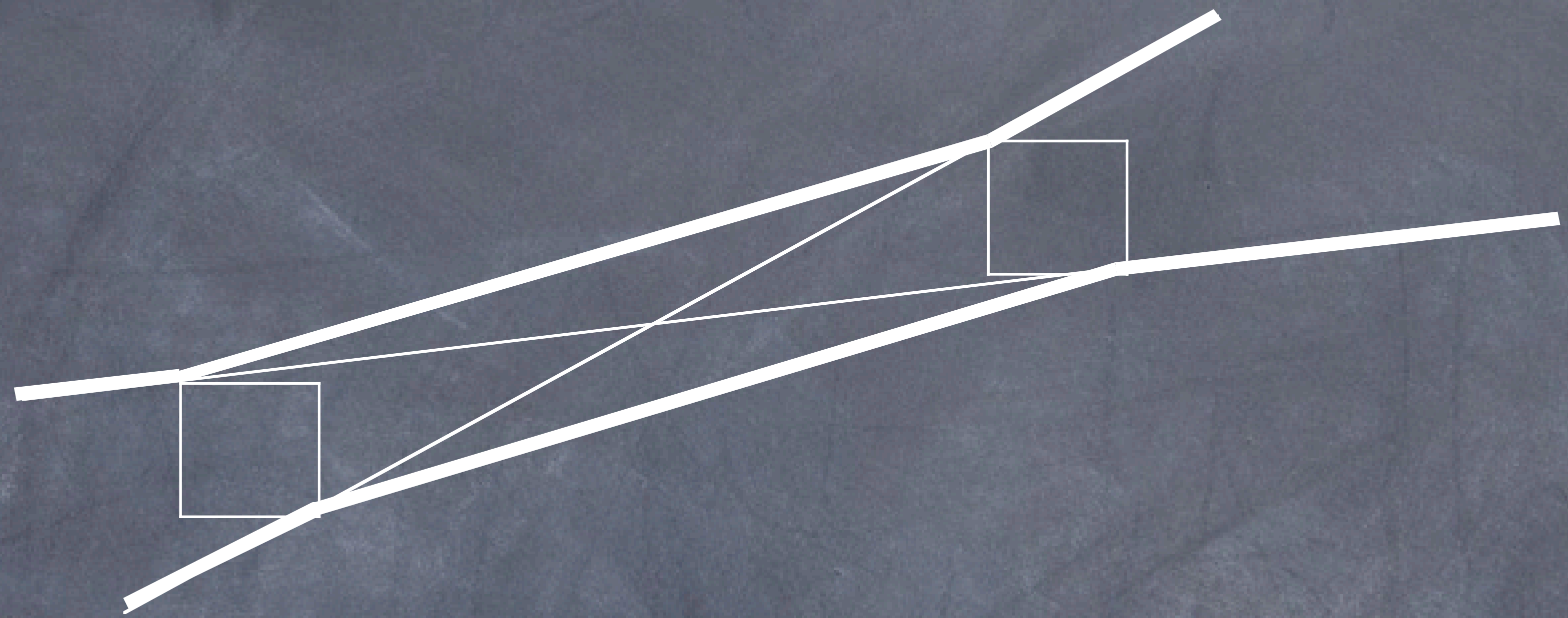
Assume that a disk cuts an nxn grid, resulting into a digital disk segment (and a complementary set). What is the asymptotic order of the number of different disk segments contained in an nxn grid?

Problem 10: Elliptic segment approximations



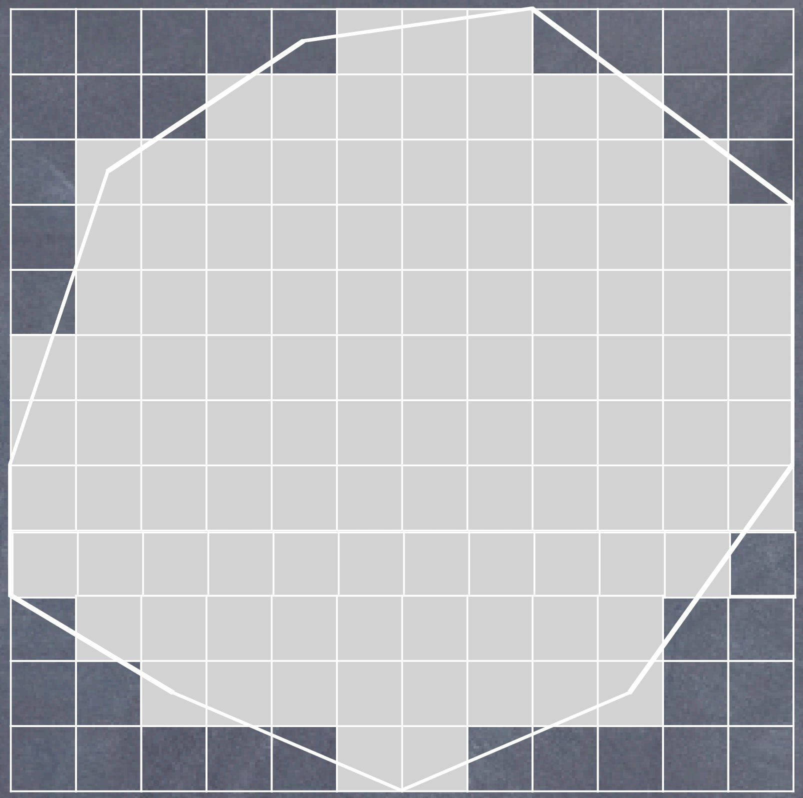
Assume a simple 1-curve of 2-cells (grid squares). Segment this curve into a consecutive sequence of maximum-length elliptic segments (efficient algorithm?). Analyse its performance for perimeter estimation (length measurement).

Problem 11: Envelope problem



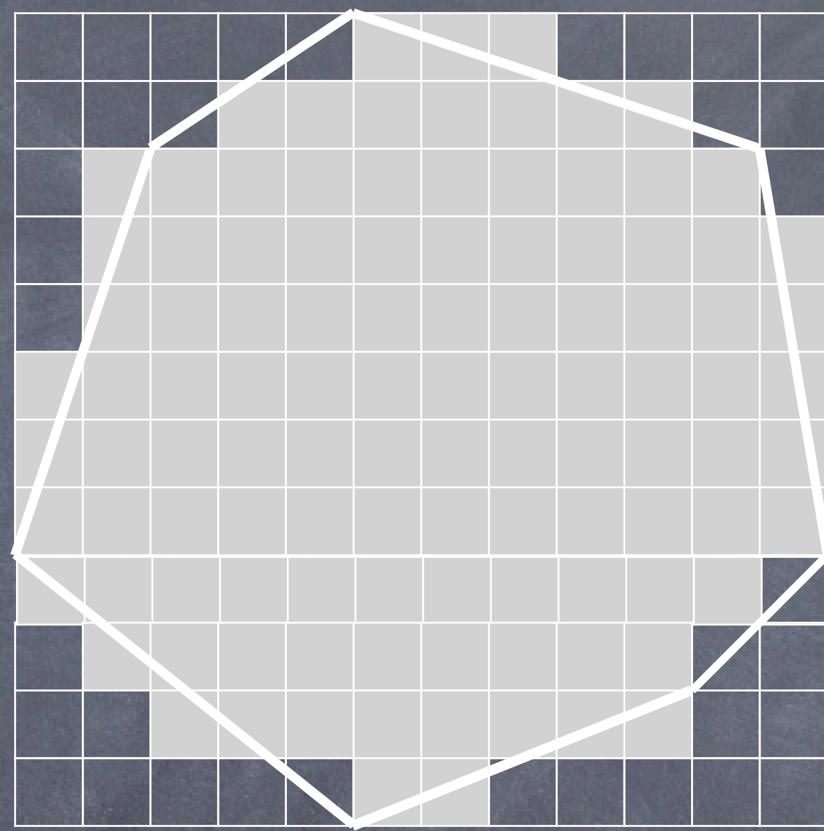
Is it possible to define digital convexity in 2D using only the envelope between (any) two grid squares contained in the set? If so, can this be extended to 3D?

Problem 12: Encoding of digital convex polygons



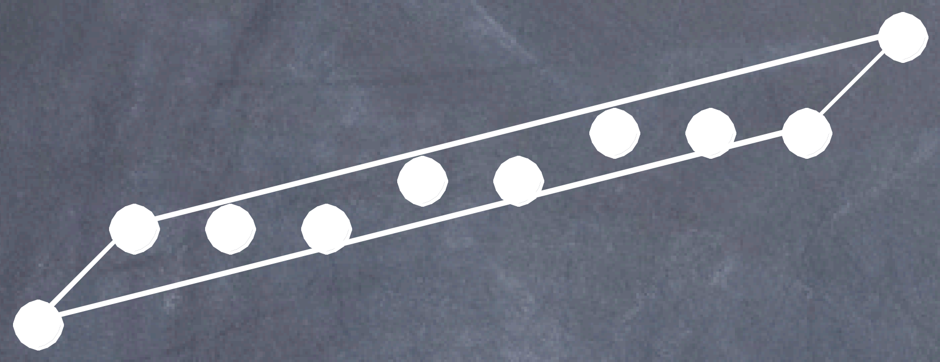
Describe an encoding scheme which requires $O(n^{2/3})$ per coded digital convex polygon, contained in an nxn grid.

Problem 13: maximum number of DSSs



Assume a DSS segmentation for a convex set contained in an nxn grid. What is the maximum number of DSSs? (in figure: 4-DSSs)

Problem 14: vertices of convex hull of DSS



Consider 8-DSS of length n. What is the number of vertices of convex hull of it.
Known: upper bound is O(log n) (J. Koplowitz...)
Open: also minimum upper bound?