Lonely Runner and view obstruction

presented by I. Sivignon during DGCI 2016

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Lonely Runner Conjecture

From Wills (1967) and Cusick (1973)

https://en.wikipedia.org/wiki/Lonely_runner_conjecture

Statement

- k runners, with different (but constant) speeds $v_i \in \mathbb{Z}^+$
- run on a circular track of length 1

$\Rightarrow$ *There is a time t where all runners are at distance $\geq \frac{1}{k+1}$ from start point for any set of speeds v_i*

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What is kown...

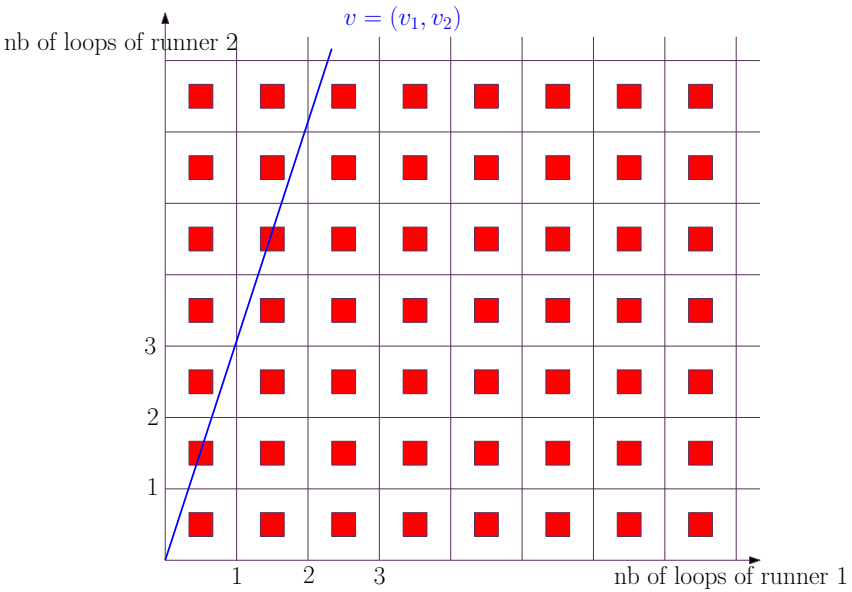
- some examples are tight (there is no t for which all runners are at a distance $> \frac{1}{k+1}$): $(1, 2, ...k) \forall k$, and also $(1, 3, 4, 7)$, $(1, 3, 4, 5, 9)$, etc.
- proofs for $k = 1, 2, 3...7$ (ad-hoc proof for each value of k)

Lonely Runner $\hookrightarrow$ view obstruction

Rewriting as a view obstruction problem

There exist integers $n_1, \dots n_k$ so that there exist a t with:

$$n_i + \frac{1}{k+1} \leq ||v_i t|| \leq n_i + \frac{k}{k+1}$$



- red squares of side $\frac{1}{k+1}$ in dimension k
- v = k-dimensionnal vector of speeds $\hookrightarrow$ blue line
- "time goes by along the blue line"
- $\Rightarrow$ prove that for any v, the blue line crosses a red square.

Some references

Applications

- ### Diophantine approximation

  J.M. Wills, Zwei Sätze ubër inhomogene diophantische Approximation von Irrationalzahlen, Monatsch. Math. 71 (1967) 263-269

- ### View obstructions

  T. W. Cusick (1973). "View-Obstruction problems". Aequationes Math. 9 (23): 165-170

- ### Distance graphs

  Eggleton, R. B., Erdös, P., Skilton, D. K. (1990). Colouring prime distance graphs. Graphs and Combinatorics, 6(1), 17-32.

- ### Nowhere zero flows

  W. Bienia, L. Goddyn, P. Gvozdjak, A. Sebö, M. Tarsi, Flows, View Obstructions, and the Lonely Runner, Journal of Combinatorial Theory, Series B, Volume 72, Issue 1, January 1998, Pages 1-9

- ### Regular colorings

  X. Zhu, Circular chromatic number of distance graphs with distance sets of cardinality 3, J. Graph Theory 41 (2002) 195-207