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**Multigrid Convergent Surface Area Estimation**

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A '3D object' is modeled by a regular solid, which is defined to be a simply-connected compact set having a measurable surface area. Assume that such a regular solid is digitized in a regular orthogonal grid with grid constant $1/r$. Specify a method for surface area estimation such that (i) it is defined for digitized regular solids, (ii) the estimated surface area will converge to the true value for $r \to \infty$ (provide a convergence theorem, i.e. a mathematical proof for a non-trivial class of regular solids), and (iii) there is an efficient algorithm implementing this method for digitized regular solids.

Until now there exist partial solutions (convergence theorem, but no algorithm, or experimental convergence but no convergence theorem), and the stated open problem would be solved with a method which combines a convergence theorem and an efficient algorithmic implementation.

Additional subproblem: Analyze the convergence speed $f(r)$ of the surface area estimation method.

**Definition:** Let $\mathcal{F}$ be the family of regular solids $S$ in 3D space, and $dig_r(S)$ a digital image of set $S$, defined by a digitization mapping $dig_r$. The surface area $A$ is defined for all sets in thi family $\mathcal{F}$. An estimator $E_A$ is *multigrid convergent* for this family $\mathcal{F}$ and the chosen digitization model $dig_r$ iff there is a grid resolution $r_S > 0$ for any set $S \in \mathcal{F}$ such that the estimator value $E_A(dig_r(S))$ is defined for any grid resolution $r \geq r_S$, and

$$|E_A(dig_r(S)) \ - \ A(S)| \leq f(r)$$

for a function $f$ defined for real numbers, having positive real values only, and converging toward 0 if $r \to \infty$. The function $f$ specifies the *convergence speed*.

# References

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