The Convex Skull Problem in Computational and Digital Geometry

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In digital image processing we are often concerned with developing specialized algorithms that are dealing with the manipulation of shapes. A very classical and widely studied approach is the computation of the convex hull of an object. However, most of these studies focus only on exterior approaches for the computation of convexity, i.e., they are looking for the smallest convex set of points including a given shape.

The problem can be presented as follows: given a non-convex polygon, how to extract the maximum area subset included in that polygon ? In [2], Goodman calls this problem the *potato-peeling problem*. More generally, Chang and Yap [1] define the polygon *inclusion* problem class $Inc(\mathcal{P}, \mathcal{Q}, \mu)$: given a general polygon $P \in \mathcal{P}$, find the μ-largest $Q \in \mathcal{Q}$ contained in P, where $\mathcal{P}$ is a family of polygons, $\mathcal{Q}$ the set of solutions and μ a real function on $\mathcal{Q}$ elements such that

$$\forall Q' \in \mathcal{Q}, \quad Q' \subseteq Q \Rightarrow \mu(Q') \leq \mu(Q). \tag{1}$$

The maximum area convex subset (or convex skull) is an inclusion problem where $\mathcal{Q}$ is the family of convex sets and μ gives the area of a solution Q in $\mathcal{Q}$.

Some results in Computational Geometry :

- the convex skull of a polygon with n vertices can be computed in $O(n^7)$ [1];
- if we consider an ortho-convex skull (*i.e.* an $hv-$convex polygon) in a ortho-polygons (polygons with edges parallel to the axis), the computational cost is $O(n^2)$ [3]. In other words, we can extract the largest $hv-$convex contained in a digital object in $O(n^2)$ where n is bounded by the number of pixels of the object boundary.

Some open problems:

- is it possible to reduce the $O(n^7)$ bound in the general case ?
- What is the computational cost if we consider the digital convex skull of a digital object ?
- Are there heuristics to obtain an approximated solution ?

References

[1] J. S. Chang and C. K. Yap. A polynomial solution for the potato-peeling problem. *Discrete & Computational Geometry*, 1:155–182, 1986.

[2] J. E. Goodman. On the largest convex polygon contained in a non-convex $n-$gon or how to peel a potato. *Geometricae Dedicata*, 11:99–106, 1981.

[3] D. Wood and C. K. Yap. The orthogonal convex skull problem. *Discrete and Computational Geometry*, 3:349–365, 1988.